# CHAPTER 9
# Testing a Claim

## 9.2
## Tests About a Population Proportion

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Starnes, Tabor, Yates, Moore

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## Tests About a Population Proportion

**Learning Objectives**

After this section, you should be able to:

- STATE and CHECK the Random, 10%, and Large Counts conditions for performing a significance test about a population proportion.
- PERFORM a significance test about a population proportion.
- INTERPRET the power of a test and DESCRIBE what factors affect the power of a test.
- DESCRIBE the relationship among the probability of a Type I error (significance level), the probability of a Type II error, and the power of a test.



## Carrying Out a Significance Test

Recall our basketball player who claimed to be an 80% free-throw shooter. In an SRS of 50 free-throws, he made 32. His sample proportion of made shots, 32/50 = 0.64, is much lower than what he claimed.

Does it provide convincing evidence against his claim?

To find out, we must perform a significance test of

$H_0$ p = 0.80
$H_a$ p < 0.80

where p = the actual proportion of free throws the shooter makes in the long run.

## Carrying Out a Significance Test for a Proportion

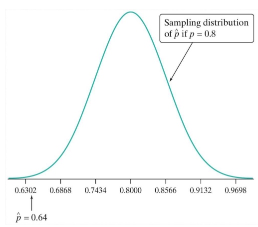
Conditions: same as Chapter 8 (intervals)

What are they?

## Carrying Out a Significance Test

If the null hypothesis $H_0$ p = 0.80 is true, then the player's sample proportion of made free throws in an SRS of 50 shots would vary according to an approximately Normal sampling distribution with mean

$$\mu_{\hat{p}} = p = 0.80 \text{ and standard deviation } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.8)(0.2)}{50}} = 0.0566$$




## Carrying Out a Significance Test

A significance test uses sample data to measure the strength of evidence against $H_0$. Here are some principles that apply to most tests:

• The test compares a statistic calculated from sample data with the value of the parameter stated by the null hypothesis.

• Values of the statistic far from the null parameter value in the direction specified by the alternative hypothesis give evidence against $H_0$

## Carrying Out a Significance Test

The test statistic says how far the sample result is from the null parameter value, and in what direction, on a standardized scale. You can use the test statistic to find the P-value of the test.

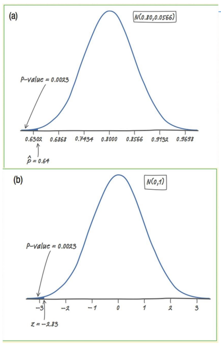
In our free-throw shooter example, the sample proportion 0.64 is pretty far below the hypothesized value $H_0$ p = 0.80.

Standardizing, we get

$$\text{test statistic} = \frac{\text{statistic - parameter}}{\text{standard deviation of statistic}}$$

$$z = \frac{0.64 - 0.80}{0.0566} = -2.83$$

## Carrying Out a Significance Test




The shaded area under the curve in (a) shows the P-value. (b) shows the corresponding area on the standard Normal curve, which displays the distribution of the Z test statistic.

Using Table A, we find that the P-value is $P(z \le -2.83) = 0.0023$.

So if $H_0$ is true, and the player makes 80% of his free throws in the long run, there's only about a 2-in-1000 chance that the player would make as few as 32 of 50 shots.

## The One-Sample z-Test for a Proportion

The four-step process is ideal for organizing our work.

**Significance Tests: A Four-Step Process**

**State:** What *hypotheses* do you want to test, and at what *significance level*? Define any *parameters* you use.
**Plan:** Choose the appropriate inference *method*. Check *conditions*.
**Do:** If the conditions are met, perform *calculations*.
- Compute the **test statistic**.
- Find the ***P*-value**.

**Conclude:** Make a *decision* about the hypotheses in the context of the problem.

## The One-Sample z-Test for a Proportion

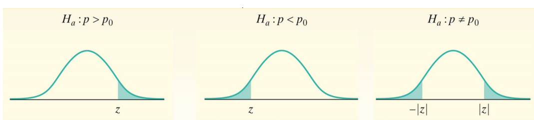
When the conditions are met—Random, 10%, and Large Counts, the sampling distribution of $\hat{p}$ is approximately Normal with mean $\mu_{\hat{p}} = p$ and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$.

The *z* statistic has approximately the standard Normal distribution when $H_0$ is true.

*P*-values therefore come from the standard Normal distribution.

## The One-Sample z-Test for a Proportion

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$




$p_o$ is the assumed value of $p$ stated in the null hypothesis.

**Why calculate z when we can just use "normalcdf"?**

Including the test statistic, z, shows better communication and is a long-established standard of statistical reporting.

Rubrics for the AP exam typically require a test statistic in addition to the p-value.

Use 1-PropZTest to check your work: hat

Enter values for $p_o$, x (# successes in your sample), n and the type of inequality for $H_a$.

Calculate and draw and see if your answers match those you obtained using the z-value.

## Two-sided Tests

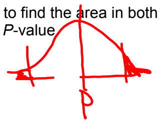
The *P*-value in a one-sided test is the area in one tail of a standard Normal distribution—the tail specified by *Ha*.

In a two-sided test, the alternative hypothesis has the form

$$H_a : p \neq p_0.$$

The *P*-value in such a test is the probability of getting a sample proportion as far as or farther from $p_0$ in either direction than the observed value of *p-hat*.

As a result, you have to find the area in both tails of a standard Normal distribution to get the *P*-value.



According to the Centers for Disease Control and Prevention (CDC) Web site, 50% of high school students have never smoked a cigarette. Taeyeon wonders whether this national result holds true in his large, urban high school. For his AP® Statistics class project, Taeyeon surveys an SRS of 150 students from his school. He gets responses from all 150 students, and 90 say that they have never smoked a cigarette. What should Taeyeon conclude? Give appropriate evidence to support your answer.

significance level was stated, we will use $\alpha = 0.05$.

**PLAN:** If conditions are met, we'll do a one-sample *z* test for the population proportion *p*.

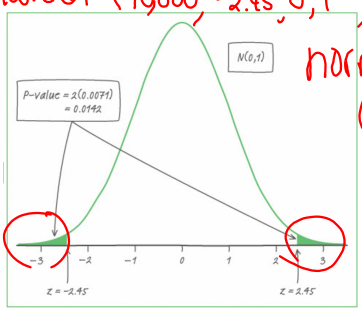
$H_0$: p= 0.5   $H_a$: p ≠ 0.5

- *Random:* Taeyeon surveyed an SRS of 150 students from his school.
  - ○ 10%: It seems reasonable to assume that there are at least 10(150) = 1500 students in a large high school.
- *Large Counts:* Assuming $H_0$: $p = 0.50$ is true, the expected counts of smokers and nonsmokers in the sample are $np_0 = 150(0.50) = 75$ and $n(1 - p_0) = 150(0.50) = 75$. Because both of these values are at least 10, we should be safe doing Normal calculations.

**DO:** The sample proportion is = 90/150 = 0.60.

- *Test statistic* $Z = \frac{.6 - .5}{\sqrt{\frac{.5(.5)}{150}}} = 2.45$
- *P-value:* The graph below displays the *P*-value as an area under the standard Normal curve for this two-sided test. To compute this *P*-value, we find the area in one tail and double it. Table A gives $P(z \geq 2.45) = 0.0071$ (the right-tail area). So the desired *P*-value is 2(0.0071) = 0.0142.

normal.cdf (-10000, -2.45, 0, 1)




normalcdf (2.45, 10,000, 0,1)

*Using technology:* The calculator's 1-PropZTest gives $z = 2.449$ and *P*-value = 0.0143.

**CONCLUDE:** Because our *P*-value, 0.0143, is less than $\alpha = 0.05$, we reject $H_0$. We have convincing evidence that the proportion of all students at Taeyeon's school who would say they have never smoked differs from the national result of 0.50.

## Why Confidence Intervals Give More Information

Calculate the 95% confidence interval for the proportion of all students at Taeyeon's school who would say they have never smoked:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$.6 \pm 1.96\sqrt{\frac{.6(.4)}{150}}$$

$$.6 \pm .078$$

$$(.522, .678)$$

Compare the formulas:

$$H_o: p = .5$$

The confidence interval and the significance test give similar but not identical information for a 2-sided test (but NOT for a one-sided test.)

The 95% confidence interval gives an approximate interval of $p_o$'s that would NOT be rejected by a two-sided test at the $\alpha = 0.05$ level.

## Type II Error and the Power of a Test

In hypothesis testing, Power is defined as the probability of rejecting the null hypothesis given that the null hypothesis is false.

$$\alpha = P(Type\ I\ error) = P(rejecting\ H_o\ |H_o\ is\ true)$$

$$\beta = P(Type\ II\ error) = P(failing\ to\ reject\ H_o|H_o\ is\ false)$$

$$Power = 1 - \beta = P(rejecting\ H_o\ |H_o\ is\ false)$$

## Type II Error and the Power of a Test

The significance level of a test is the probability of reaching the wrong conclusion when the null hypothesis is true.

The power of a test to detect a specific alternative is the probability of reaching the right conclusion when that alternative is true.

We can just as easily describe the test by giving the probability of making a Type II error (sometimes called $\beta$)

**Power and Type II Error**

The power of a test against any alternative is 1 minus the probability of a Type II error for that alternative; that is, power = $1 - \beta$

## Tests About a Population Proportion

**Section Summary**

In this section, we learned how to…

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