In Chapter 2, we studied the effects of linear transformations on the shape, center, and spread of a distribution of data. Recall:

- Adding (or subtracting) a constant, a, to each observation:
  Adds a to measures of center and location.
  Does not change the shape or measures of spread.

- Multiplying (or dividing) each observation by a constant, b:
  Multiplies (divides) measures of center and location by b.
  Multiplies (divides) measures of spread by |b|.
  Does not change the shape of the distribution.

Jan 6-11:03 AM

Multiplying a Random Variable by a Constant

Effect on a Random Variable of Multiplying (Dividing) by a Constant

Multiplying (or dividing) each value of a random variable by a number *b*:

- Multiplies (divides) measures of center and location (mean, median, quartiles, percentiles) by *b*.
- Multiplies (divides) measures of spread (range, *IQR*, standard deviation) by |*b*|.
- Does not change the shape of the distribution.

As with data, if we multiply a random variable by a negative constant b, our common measures of spread are multiplied by |b|.

Jan 6-12:51 PM

Adding a Constant to a Random Variable

Effect on a Random Variable of Adding (or Subtracting) a Constant

Adding the same number a (which could be negative) to each value of a random variable:

- Adds a to measures of center and location (mean, median, quartiles, percentiles).
- Does not change measures of spread (range, IQR, standard deviation).
- Does not change the shape of the distribution.

Jan 6-11:48 AM

Linear Transformations

Effect on a Linear Transformation on the Mean and Standard Deviation

If $Y = a + bX$ is a linear transformation of the random variable *X*, then

•The probability distribution of *Y* has the same shape as the probability distribution of *X*.

•$\mu_Y = a + b\mu_X$.

•$\sigma_Y = |b|\sigma_X$ (since *b* could be a negative number).

Linear transformations have similar effects on other measures of center or location (median, quartiles, percentiles) and spread (range, IQR).

Whether we're dealing with data or random variables, the effects of a linear transformation are the same.

Note: These results apply to both discrete and continuous random variables.

Jan 6-11:12 AM

Jan 6-12:16 PM

PROBLEM: One brand of bathtub comes with a dial to set the water temperature. When the "babysafe" setting is selected and the tub is filled, the temperature X of the water follows a Normal distribution with a mean of 34°C and a standard deviation of 2°C.

(a) Define the random variable Y to be the water temperature in degrees Fahrenheit when the dial is set on "babysafe." Find the mean and standard deviation of Y.

(b) According to Babies R Us, the temperature of a baby's bathwater should be between 90°F and 100°F. Find the probability that the water temperature on a randomly selected day when the "babysafe" setting is used meets the Babies R Us recommendation. Show your work.

Jan 6-12:11 PM

a) $\mu_x = 34 \quad \sigma_x = 2$

$\mu_y = 34 \cdot \frac{9}{5} + 32 = 93.2$

$\sigma_y = 2\left(\frac{9}{5}\right) = 3.6$

b) $N(93.2, 3.6)$

$P(90 \le X \le 100)$

normalcdf(90, 100, 93.2, 3.6)

≈ 78%

Jan 8-1:36 PM

The Mean of the Sum

For any two random variables X and Y, if T = X + Y, then the expected value of T is

$$E(T) = \mu_T = \mu_X + \mu_Y$$

In general, the mean of the sum of several random variables is the sum of their means.

Note that for this to be meaningful, X and Y must be independent.

Jan 6-11:49 AM

Variance of a Sum

For any two independent random variables X and Y, if T = X + Y, then the variance of T is

Var(T) = Var(X) + Var(Y)

$$\sigma_T^2 = \sigma_X^2 + \sigma_Y^2$$

NOTE: $\sigma_T \neq \sigma_x + \sigma_Y$

Jan 6-11:54 AM

You can add variances only if the two random variables are independent, and you can NEVER add standard deviations!

Jan 7-1:33 PM

Mean of the Difference of Random Variables

For any two random variables *X* and *Y*, if *D* = *X* - *Y*, then the expected value of *D* is

$$E(D) = \mu_D = \mu_X - \mu_Y$$

In general, the mean of the difference of several random variables is the difference of their means. *The order of subtraction is important!*

Variance of the Difference of Random Variables

For any two random variables *X* and *Y*, if *D* = *X* - *Y*, then the variance of *D* is

$$\sigma_D^2 = \sigma_X^2 + \sigma_Y^2$$

In general, the variance of the difference of two independent random variables is the sum of their variances.

Jan 6-12:16 PM

If a random variable is Normally distributed, we can use its mean and standard deviation to compute probabilities.

- Any sum or difference of independent Normal random variables is also Normally distributed.

Jan 7-1:35 PM

Mr. Starnes likes between 8.5 and 9 grams of sugar in his hot tea. Suppose the amount of sugar in a randomly selected packet follows a Normal distribution with mean 2.17 g and standard deviation 0.08 g. If Mr. Starnes selects 4 packets at random, what is the probability his tea will taste right?

Let X_i = amt of sugar in any 1 packet

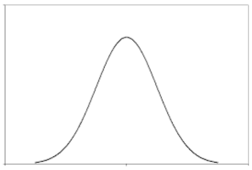
Let T = amt of sugar in 4 packets

μ_T =

Var(T) =

σ_T =

Jan 7-1:35 PM



Jan 7-1:44 PM

Jan 5-10:48 AM