CHAPTER 6
Random Variables

Bedford Freeman Worth Publishers

Discrete and Continuous Random Variables

Learning Objectives

After this section, you should be able to:

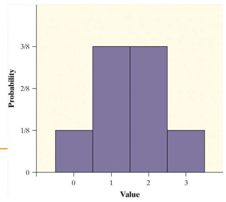
- COMPUTE probabilities using the probability distribution of a discrete random variable.
- CALCULATE and INTERPRET the mean (expected value) of a discrete random variable.
- CALCULATE and INTERPRET the standard deviation of a discrete random variable.
- COMPUTE probabilities using the probability distribution of certain continuous random variables.

Random Variables and Probability Distributions

A **probability model** describes the possible outcomes of a chance process and the likelihood that those outcomes will occur.

Consider tossing a fair coin 3 times.
Define X = the number of heads obtained

X = 0: TTT
X = 1: HTT THT TTH
X = 2: HHT HTH THH
X = 3: HHH



A **random variable** takes numerical values that describe the outcomes of some chance process. The **probability distribution** of a random variable gives its possible values and their probabilities.

Example:

- If a fair coin is tossed 3 times, what is the probability that we get at least one head?
- $P(X \geq 1) =$

- Complete the alternate example in your notes.

Random Variables

- A variable whose value is determined by chance
- Denoted with capital letters, usually X, Y or Z, but can be any letter.
- Example: Suppose you buy a ticket to a raffle whose prizes are \$5, \$50, \$100 or nothing. The random variable, X, could represent the value of the prize that you win. Possible values of X are 0, 5, 50 and 100.

Create the probability model for X:

Summary

- What is a random variable? Give some examples.

- What is a probability distribution?

Discrete Random Variable

A variable whose values can be listed with gaps between values (but the list does not have to end).

The probability distribution of a discrete random variable, X, lists the values, x_i, and their probabilities, p_i

Example: Let X = the number of rolls of a fair die needed to get a 6.

Example: Compare foot length and shoe size. Which is discrete? The other is continuous.

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When analyzing discrete random variables, we follow the same strategy we used with quantitative data –describe the shape, center, and spread, and identify any outliers.

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Mean (Expected Value) of a Discrete Random Variable

The mean of any discrete random variable is an average of the possible outcomes, with each outcome weighted by its probability

The mean of X, the number of heads in 3 tosses of a fair coin, is

Formula for the mean of a discrete random variable:

$$\mu_X = E(X) = x_1p_1 + x_2p_2 + x_3p_3 \ldots$$

$$= \sum x_i p_i$$

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Mean (Expected Value) of a Discrete Random Variable

A baby's Apgar score is the sum of the ratings on each of five scales, which gives a whole-number value from 0 to 10.

Let X = Apgar score of a randomly selected newborn

Value:	0	1	2	3	4	5	6	7	8	9	10
Probability:	0.001	0.006	0.007	0.008	0.012	0.020	0.038	0.099	0.319	0.437	0.053

Compute the mean of the random variable X. Interpret this value in context.

The mean Apgar score of a randomly selected newborn is 8.128. This is the average Apgar score of many, many randomly chosen babies.

AP Common Error

Do not round the mean of a random variable to an integer value. The mean value does NOT have to be equal to one of the possible values of the variable.

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Standard Deviation of a Discrete Random Variable

Since we use the mean as the measure of center for a discrete random variable, we use the standard deviation as our measure of spread. The definition of the **variance of a random variable** is similar to the definition of the variance for a set of quantitative data.

$$Var(X) = \sigma_X^2 = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + (x_3 - \mu_X)^2 p_3 + \ldots$$
$$= \sum (x_i - \mu_X)^2 p_i$$

To get the standard deviation of a random variable , take the square root of the variance.

The standard deviation of a random variable gives an indication of

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Standard Deviation of a Discrete Random Variable

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Compute and interpret the standard deviation of the random variable X

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Continuous Random Variables

Discrete random variables commonly arise from situations that involve counting something. Situations that involve measuring something often result in a **continuous random variable**

A **continuous random variable** X takes on all values in an interval of numbers. The probability distribution of X is described by a density curve. The probability of any event is the area under the density curve and above the values of X that make up the event.

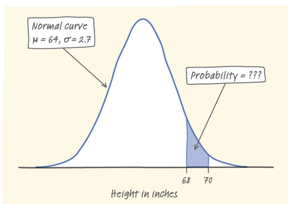
A continuous random variable Y has infinitely many possible values. All continuous probability models assign probability 0 to every individual outcome. Only intervals of values have positive probability.

Example: Normal probability distributions

The heights of young women closely follow the Normal distribution with mean μ = 64 inches and standard deviation σ = 2.7 inches.

Now choose one young woman at random. Call her height Y
If we repeat the random choice very many times, the distribution of values of Y is the same Normal distribution that describes the heights of all young women.

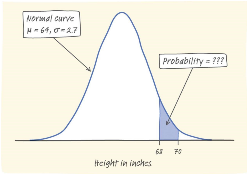
Problem: What's the probability that the chosen woman is between 68 and 70 inches tall?



Example: Normal probability distributions

Step 1: State the distribution and the values of interest.

The height Y of a randomly chosen young woman has the N(64, 2.7) distribution. We want to find P(68 ≤ Y ≤ 70).



Step 2: Perform calculations—show your work!

The standardized scores for the two boundary values are

$$z = \frac{68-64}{2.7} = 1.48$$

$$z = \frac{70-64}{2.7} = 2.22$$

$$P(1.48 \le Z \le 2.22) = P(Z \le 2.22) - P(Z \le 1.48)$$
$$= 0.9868 - 0.9306 = 0.0562$$

Discrete and Continuous Random Variables

Section Summary

In this section, we learned how to…

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