CHAPTER 5
Probability: What Are the Chances?
5.3 Conditional Probability and Independence

The Practice of Statistics, 5th Edition
Starnes, Tabor, Yates, Moore

Bedford Freeman Worth Publishers

Conditional Probability and Independence

Learning Objectives

After this section, you should be able to:

- CALCULATE and INTERPRET conditional probabilities.
- USE the general multiplication rule to CALCULATE probabilities.
- USE tree diagrams to MODEL a chance process and CALCULATE probabilities involving two or more events.
- DETERMINE if two events are independent.
- When appropriate, USE the multiplication rule for independent events to COMPUTE probabilities.

What is Conditional Probability?

The probability we assign to an event can change if we know that some other event has occurred.

Suppose we know that event A has happened. Then the probability that event B happens given that event A has happened is denoted by P(B | A)

In a two-way table, conditional probability is easier to see:

2 psychologists surveyed 478 4th, 5th and 6th graders in Michigan about their primary goals: to get good grades, to be popular, or to be good at sports. Here are the results based on gender:

Sex	Goals: Grades	Popular	Sports	Total
Boy	117	50	60	227
Girl	130	91	30	251
Total	247	141	90	478

How do the goals of boys and girls compare?

If we choose a student at random, what is the probability of choosing a girl? P(girl) =

If we choose a person at random, what is the probability that it will be a girl who prefers sports?

P(girl ∩ sports) =

What is the probability that if a chosen student is a girl, that she will prefer sports?

P(sports|girl) =

Let's look at the math:

$$P(\text{sports}|\text{girl}) = \frac{30}{251}$$

$$P(\text{girl} \cap \text{sports}) = \frac{30}{478} \qquad P(\text{girl}) = \frac{251}{478}$$

$$P(\text{sports}|\text{girl}) = \frac{P(\text{girl} \cap \text{sports})}{P(\text{girl})}$$

Click

Calculating Conditional Probabilities

The probability of A given B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The probability of B given A:

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

first given second

both

second

The General Multiplication Rule

Doing some algebra with the last equations we get:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B) = P(A|B) \cdot P(B)$

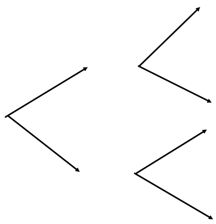
and

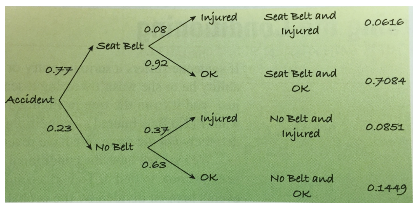
$P(A \cap B) = P(B|A) \cdot P(A)$

In words, this rule says that for both of two events to occur, first one must occur, and then given that the first event has occurred, the second must occur.

A Maryland highway safety study foud that in 77% of all accidents, the driver was wearing a seat belt. 92% of those drivers escaped serious injury, but only 63% of those not wearing seat belts were so fortunate. Overall, what is the probability that a driver involved in an accident was seriously injured?

This problem is best modeled with a tree diagram.





P(A|B) $\qquad$ $P(A \cap B)$

Sums to 1

A computer company makes desktop and laptop computers at factories in three states—California, Texas, and New York. The California factory produces 40% of the company's computers, the Texas factory makes 25%, and the remaining 35% are manufactured in New York. Of the computers made in California,75% are laptops. Of those made in Texas and New York, 70% and 50%, respectively, are laptops. All computers are first shipped to a distribution center in Missouri before being sent out to stores. Suppose we select a computer at random from the distribution center.

1. Construct a tree diagram to represent this situation.

2. Find the probability that the computer is a laptop. Show your work.

3. Given that a laptop is selected, what is the probability that it was made in California?

Reversing the Conditioning:

If someone suffers a severe injury in MD, what is the probability he or she wasn't wearing a seat belt?

P(No belt | injured) is not on the diagram.

Use the formula

$$P(\text{No belt} \mid \text{Injured}) = \frac{P(\text{No belt AND Injured})}{P(\text{Injured})}$$

Example: Late for School
Shannon hits the snooze bar on her alarm clock on 60% of school days. If she doesn't hit the snooze bar, there is a 0.90 probability that she makes it to class on time. However, if she hits the snooze bar, there is only a 0.70 probability that she makes it to class on time.

(a) Use a tree diagram to represent this situation.

1 On a randomly chosen day, what is the probability that Shannon is late for class?

2 Let H be the event that Shannon hits the snooze bar and L be the event that Shannon is late for school.

What is $P(H|L)$?

3 Let H be the event that Shannon hits the snooze bar and L be the event that Shannon is late for school.

What is $P(L|H)$?

4 What is $P(H \cap L)$?

5 What is $P(H \cup L)$?

Conditional Probability and Independence

Two events A and B are **independent** if the occurrence of one event does not change the probability that the other event will happen. In other words, events A and B are independent if
$P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$.

When events A and B are independent, we can simplify the general multiplication rule since $P(B \mid A) = P(B)$.

Multiplication rule for independent events
If A and B are independent events, then the probability that A and B both occur is
$P(A \cap B) = P(A) \cdot P(B)$

What does it mean for events to be independent?

Testing for independence

Back to our study about the goals of 4th, 5th and 6th grade students:

Is choosing success in sports as a goal independent of gender?

If it were, then

P(sports) would = P(sports|girl), and

P(sports) would = P(sports|boy)

What about grades?

Pull

Multiplication Rule for Independent Events

Following the Space Shuttle Challenger disaster, it was determined that the failure of O-ring joints in the shuttle's booster rockets was to blame. Under cold conditions, it was estimated that the probability that an individual O-ring joint would function properly was 0.977.

Assuming O-ring joints succeed or fail independently, what is the probability all six would function properly?

P(joint 1 OK and joint 2 OK and joint 3 OK and joint 4 OK and joint 5 OK and joint 6 OK)

Pull

The multiplication rule below holds if A and B are independent, but not otherwise.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

The addition rule below holds if A and B are mutually exclusive, but not otherwise.

$$P(A \text{ or } B) = P(A) + P(B)$$

	Gender		
Dominant Hand	Male	Female	**Total**
Right	39	51	**90**
Left	7	3	**10**
Total	**46**	**54**	**100**

PROBLEM: Are the events "male" and "left-handed" independent? Justify your answer.

In baseball, a perfect game is when a pitcher doesn't allow any hitters to reach base in all nine innings. Historically, pitchers throw a perfect inning—an inning where no hitters reach base—about 40% of the time. So, to throw a perfect game, a pitcher needs to have nine perfect innings in a row.

Problem: What is the probability that a pitcher throws nine perfect innings in a row, assuming the pitcher's performance in an inning is independent of his performance in other innings?

The opposite of "none" is "at least one".

P(at least one) = 1 - P(none)

The First Trimester Screen is a noninvasive test given during the first trimester of pregnancy to determine if there are specific chromosomal abnormalities in the fetus. According to a study published in the *New England Journal of Medicine* in November 2005), approximately 5% of normal pregnancies will receive a positive result.
Problem: If 100 women with normal pregnancies are tested with the First Trimester Screen, what is the probability that at least 1 woman will receive a positive result?

What can go wrong?

- Beware of probabilities that don't add up to 1.
- Don't add probabilities if the events are not disjoint.
- Only multiply probabilities if events are independent.
- Don't confuse disjoint and independent.
- Don't reverse conditioning: In general, $P(A|B) \neq P(B|A)$.
- Pay attention to "with replacement" and "without replacement".

Conditional Probabilities and Independence

Section Summary

In this section, we learned how to…

- CALCULATE and INTERPRET conditional probabilities.
- USE the general multiplication rule to CALCULATE probabilities.
- USE tree diagrams to MODEL a chance process and CALCULATE probabilities involving two or more events.
- DETERMINE if two events are independent.
- When appropriate, USE the multiplication rule for independent events to COMPUTE probabilities.