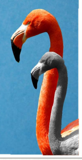
CHAPTER 5
Probability: What Are the Chances?

5.2
Probability Rules



The Practice of Statistics, 5th Edition
Starnes, Tabor, Yates, Moore

Bedford Freeman Worth Publishers

Probability Rules

Learning Objectives

After this section, you should be able to:

- DESCRIBE a probability model for a chance process.
- USE basic probability rules, including the complement rule and the addition rule for mutually exclusive events.
- USE a two-way table or Venn diagram to MODEL a chance process and CALCULATE probabilities involving two events.
- USE the general addition rule to CALCULATE probabilities.

Probability Models

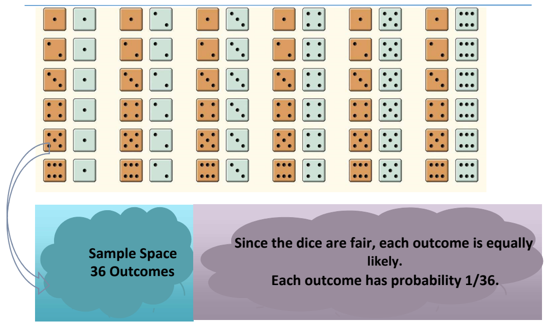
In Section 5.1, we used simulation to imitate chance behavior. Fortunately, we don't have to always rely on simulations to determine the probability of a particular outcome.

Descriptions of chance behavior contain two parts:

The **sample space** **S** of a chance process is the

A **probability model** is a description of some chance process that consists of two parts:

Example: Building a probability model



Event:

Sample Space:

Let A be the event of getting a 7 on the roll of 2 die.

P(A) =

Let B be the event of getting a number greater than 8.

P(B) =

Define 2 more events related to rolling 2 die and calculate their probability. Share with a partner.

Finding probability when events are equally likely is straightforward, but gets harder as the number of outcomes gets bigger.

How many outcomes are in the sample space of flipping a coin 100 times? What is the probability of the event of getting 67 heads?

Answer

Random events are not always equally likely!

If you play a lottery, you can win or not win. Are they equally likely?

Basic Rules of Probability

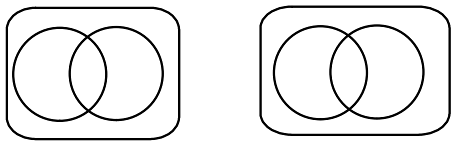
- The probability of any event is a number
- All possible outcomes together must have probabilities whose sum
- If all outcomes in the sample space are equally likely, the probability that event A occurs can be found using the formula
- The probability that an event does *not* occur is
- If two events have no outcomes in common, the probability that one or the other occurs is

What about probability when an outcome can take on any value in a range? For example, what is the probability that a randomly selected child is exactly 3 feet tall?

Answer

Mutually exclusive events:

- can never occur together
- also called disjoint
- $P(A \cup B) = 0$



Basic Rules of Probability

We can summarize the basic probability rules more concisely in symbolic form.

Basic Probability Rules

- For any event, A,
- If S is the sample space in a probability model,
- In the case of equally likely outcomes,

  P(A) =

- Complement rule:
- Addition rule for mutually exclusive events:

Imagine flipping a fair coin three times.
Describe the probability model for this chance process and use it to find the probability of getting at least 1 head in three flips.

Two-Way Tables and Probability

When finding probabilities involving two events, a two-way table can display the sample space in a way that makes probability calculations easier.

Consider the example on page 309. Suppose we choose a student at random. Find the probability that the student

(a)has pierced ears.

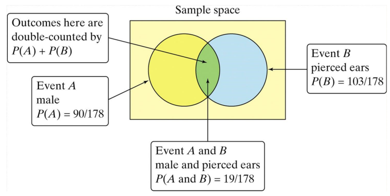
(b)is a male with pierced ears.

(c)is a male or has pierced ears.

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**

General Addition Rule for Two Events

We can't use the addition rule for mutually exclusive events unless the events have no outcomes in common.

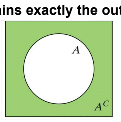


General Addition Rule for Two Events

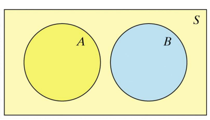
Venn Diagrams and Probability

Because Venn diagrams have uses in other branches of mathematics, some standard vocabulary and notation have been developed.

The complement A^C contains exactly the outcomes that are not in A.

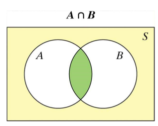


The events A and B are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.

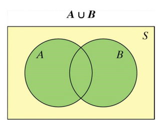


Venn Diagrams and Probability

The intersection of events A and B, $A \cap B$ is the set of all outcomes in both events A and B.



The union of events A and B, $A \cup B$ is the set of all outcomes in either event A or B.

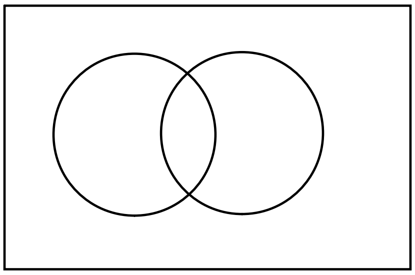


Hint: To keep the symbols straight, remember ∪ for **U**nion and ∩ for i**n**tersection.

Alternate Example: Who Owns a Home?
What is the relationship between educational achievement and home ownership? A random sample of 500 people who participated in the 2000 census was chosen. Each member of the sample was identified as a high school graduate (or not) and as a home owner (or not). Overall, 340 were homeowners, 310 were high school graduates, and 221 were both homeowners and high school graduates.

(a) Create a two-way table that displays the data.

b) Make a Venn Diagram to display the sample space



Suppose we choose a member of the sample at random. Find the probability that the member

(c) is a high school graduate.

(d) is a high school graduate and owns a home.

(e) is a high school graduate or owns a home.

Randomly select a student who took the 2010 AP Statistics exam and record the student's score. Here is the probability model:

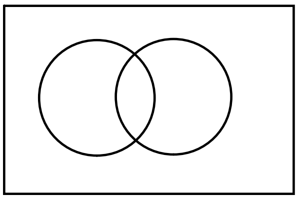
Score	1	2	3	4	5
Probability	0.233	0.183	0.235	0.224	0.125

(a) Show that this is a legitimate probability model.
(b) Find the probability that the chosen student scored 3 or better.
(c) Find the probability that the chosen student didn't get a 1.

Example: Phone Usage

According to the National Center for Health Statistics, in December 2012, 60% of US households had a traditional landline telephone, 89% of households had cell phones, and 51% had both. Suppose we randomly selected a household in December 2012.

(a) Make a two-way table that displays the sample space of this chance process.
(b) Construct a Venn diagram to represent the outcomes of this chance process.
(c) Find the probability that the household has at least one of the two types of phones.
(d) Find the probability that the household has neither type of phone.
(e) Find the probability the household has a cell phone only.



Probability Rules

Section Summary

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