CHAPTER 11
Inference for Distributions of Categorical Data

11.2
Inference for Two-Way Tables

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Inference for Two-Way Tables

Learning Objectives

After this section, you should be able to:

- $ COMPAREconditional distributions for data in a two-way table.
- $ STATE appropriate hypotheses and COMPUTE expected counts for a chi-square test based on data in a two-way table.
- $ CALCULATE the chi-square statistic, degrees of freedom, and P-value for a chi-square test based on data in a two-way table.
- $ PERFORM a chi-square test for homogeneity.
- $ PERFORM a chi-square test for independence.
- $ CHOOSE the appropriate chi-square test.

In Chapter 10, we compared the proportions of successes across 2 populations or for 2 treatments.

What if we want to compare more than 2 groups or treatments?

We will use 2-way tables and apply the Chi-square test for homogeneity.

are 2 pops similar (homogeneous)

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Comparing Distributions of a Categorical Variable

Market researchers suspect that background music may affect the mood and buying behavior of customers.

One study in a Mediterranean restaurant compared three randomly assigned treatments: no music, French accordion music, and Italian string music.

Under each condition, the researchers recorded the numbers of customers who ordered French, Italian, and other entrees.

	Type of Music			
Entree ordered	**None**	**French**	**Italian**	**Total**
French	30	39	30	**99**
Italian	11	1	19	**31**
Other	43	35	35	**113**
Total	**84**	**75**	**84**	**243**

Find the proportions or conditional distribution for each type of music:

	Music			
Entree	None	French	Italian	Total
French	0.3571	0.52	0.357	
Italian	0.131	0.013	0.226	
Other	0.5119	0.4671	0.418	
Total	1	1	1	1.00

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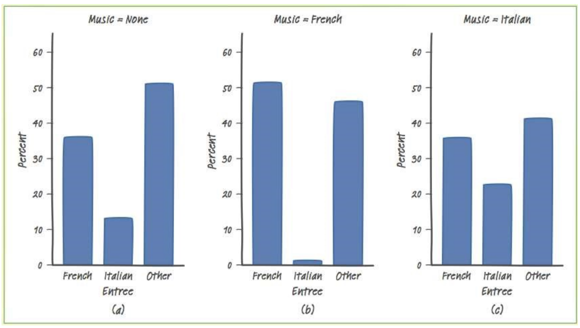
Comparing Distributions of a Categorical Variable

Problem: (b) Make an appropriate graph for comparing the conditional distributions in part (a).



Comparing Distributions of a Categorical Variable

Problem: (c) Write a few sentences comparing the distributions of entrees ordered under the three music treatments.

The type of entrée that customers buy seems to differ considerably across the three music treatments.

Orders of Italian entrees are very low (1.3%) when French music is playing but are higher when Italian music (22.6%) or no music (13.1%) is playing.

French entrees seem popular in this restaurant, as they are ordered frequently under all music conditions but notably more often when French music is playing.

For all three music treatments, the percent of Other entrees ordered was similar.

Expected Counts and the Chi-Square Statistic

A chi-square test for homogeneity begins with the hypotheses

H_0: There is no difference in the distribution of a categorical variable for several populations or treatments.

H_a: There is a difference in the distribution of a categorical variable for several populations or treatments.

We compare the observed counts in a two-way table with the counts we would expect if H_0 were true.

Expected Counts and the Chi-Square Statistic

Observed Counts

Entree ordered	None	French	Italian	Total
	Type of Music			
French	30	39	30	99
Italian	11	1	19	31
Other	43	35	35	113
Total	**84**	**75**	**84**	**243**

For each entry, the expected count is found by multiplying the row and column totals and dividing by the table total:

$\frac{84 \cdot 99}{243}$ $\frac{84 \cdot 113}{243}$

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Conditions:

Same as for Chi-square GOF:

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Calculating Chi-square:

The formula is the same:

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

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On the calculator:

Enter your two-way table data (without the totals) of your observed counts into a matrix:

[2nd] [x^{-1}] Choose EDIT, then ENTER to select ~~[A]~~. [B]

Enter the dimensions of your matrix (rows x columns).

Enter the data.

Run the χ^2-Test

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Example: P-value and conclusion

Problem: (b) Interpret the P-value from the calculator in context.

Assuming that there is no difference in the true distributions of entrees ordered in this restaurant when no music, French accordion music, or Italian string music is played, there is a 0,0011 probability of observing a difference in the distributions of entrees ordered among the three treatment groups as large or larger than the ones in this study.

Problem: (c) What conclusion would you draw? Justify your answer

Because the P-value, 0.0011, is less than our default $\alpha = 0.05$ significance level, we reject H_0. We have convincing evidence of a difference in the distributions of entrees ordered at this restaurant when no music, French accordion music, or Italian string music is played. Furthermore, the random assignment allows us to say that the difference is caused by the music that's played.

Follow-Up Analysis

The chi-square test for homogeneity allows us to compare the distribution of a categorical variable for any number of populations or treatments. If the test allows us to reject the null hypothesis of no difference, we then want to do a follow-up analysis that examines the differences in detail.

Start by examining which cells in the two-way table show large deviations between the observed and expected counts. Then look at the individual components to see which terms contribute most to the chi-square statistic.

Looking at the output, we see that just two of the nine components that make up the chi-square statistic contribute about 14 (almost 77%) of the total $\chi^2 = 18.28$.

We are led to a specific conclusion: orders of Italian entrees are strongly affected by Italian and French music

Follow-Up Analysis

```
Chi-Square Test: None, Franch, Italian

Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts

           None     French    Italian    Total
1            30         39         30       99
          34.22      30.56      34.22
          0.521      2.334      0.521

2            11          1         19       31
          10.72       9.57      10.72
          0.008      7.672      6.404

3            43         35         35      113
          39.06      34.88      39.06
          0.397      0.000      0.422

Total        84         75         84      243

Chi-Sq = 18.279, DF = 4, P-Value = 0.001
```

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Relationships Between Categorical Variables

Another common situation that leads to a two-way table is when a single random sample of individuals is chosen from a single population and then classified based on two categorical variables.

In that case, our goal is to analyze the relationship between the variables.

Our null hypothesis is that there is no association between the two categorical variables in the population of interest.

The alternative hypothesis is that there is an association between the variables.

The Chi-Square Test for Independence

The 10% and Large Counts conditions for the chi-square test for independence are the same as for the homogeneity test.

There is a slight difference in the Random condition for the two tests: a test for independence uses data from one sample but a test for homogeneity uses data from two or more samples/groups.

H_0: There is no association between two categorical variables in the population of interest.
H_a: There is an association between two categorical variables in the population of interest.

Example: Choosing the right type of chi-square test

Are men and women equally likely to suffer lingering fear from watching scary movies as children? Researchers asked a random sample of 117 college students to write narrative accounts of their exposure to scary movies before the age of 13. More than one-fourth of the students said that some of the fright symptoms are still present when they are awake. The following table breaks down these results by gender.

	Gender		
Fright symptoms?	Male	Female	Total
Yes	7	29	**36**
No	31	50	**81**
Total	**38**	**79**	**117**

Example: Choosing the right type of chi-square test

Minitab output for a chi-square test using these data is shown below.

```
Chi-Square Test: Male, Female
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts
          Male      Female      Total
1            7          29         36
         11.69       24.31
         1.883       0.906

2           31          50         81
         26.31       54.69
         0.837       0.403

Total       38          79        117
Chi-Sq = 4.028, DF = 1, P-Value = 0.045
```

Example: Choosing the right type of chi-square test

Problem: Assume that the conditions for performing inference are met.

(a) Explain why a chi-square test for independence and not a chi-square test for homogeneity should be used in this setting.

The data were produced using a single random sample of college students, who were then classified by gender and whether or not they had lingering fright symptoms.

The chi-square test for homogeneity requires independent random samples from each population.

Example: Choosing the right type of chi-square test

Problem: Assume that the conditions for performing inference are met.

(b) State an appropriate pair of hypotheses for researchers to test in this setting.

The null hypothesis is

H_0: There is no association between gender and ongoing fright symptoms in the population of college students. The alternative hypothesis is

H_a: There is an association between gender and ongoing fright symptoms in the population of college students.

Example: Choosing the right type of chi-square test

Problem: Assume that the conditions for performing inference are met.

(c) Which cell contributes most to the chi-square statistic? In what way does this cell differ from what the null hypothesis suggests?

Example: Choosing the right type of chi-square test

Problem: Assume that the conditions for performing inference are met.

(d) Interpret the P-value in context. What conclusion would you draw at $\alpha = 0.01$?

If gender and ongoing fright symptoms really are independent in the population of interest, there is a 0.045 chance of obtaining a random sample of 117 students that gives a chi-square statistic of 4.028 or higher. Because the P-value, 0.045, is greater than 0.01, we would fail to reject H_0. We do not have convincing evidence that there is an association between gender and fright symptoms in the population of college students.

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