CHAPTER 11
Inference for Distributions of Categorical Data

Chi-Square Tests for Goodness of Fit

Learning Objectives

After this section, you should be able to:

- STATE appropriate hypotheses and COMPUTE expected counts for a chi-square test for goodness of fit.
- CALCULATE the chi-square statistic, degrees of freedom, and P-value for a chi-square test for goodness of fit.
- PERFORM a chi-square test for goodness of fit.
- CONDUCT a follow-up analysis when the results of a chi-square test are statistically significant.

Chi-square Goodness-of-fit Test

How well does a set of data for one variable fit the expected values?

Chi-square test for Homogeneity

Comparing two or more populations or treatments

Chi-square test for Independence

Comparing two or more categorical variables with one population

Apr 5-3:32 PM

The Candy Man Can

Mars, Inc. makes M&M's. Here is what the company says about the color distribution of its plain M&M's:

On average 13% each of brown and red, 14% yellow, 16% green, 20% orange, 24% blue.

Your team has a sample bag of M&M's. Is the distribution of colors in your bag different from what the company says?

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How do these numbers compare to the companies claim?

Since we have multiple colors, we can't use the one proportion z test. Instead we'll use the Chi-squared Goodness of Fit.

Complete the table below:

Colors	Red	Blue	Brown	Orange	Green	Yellow
Observed values						
Expected values						

(How do we find the expected counts?)

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State your hypotheses:

H_0: the color distribution of M&M's is consistent to what the company says

H_a: the color distribution of M&M's is not consistent with what the company says

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The Chi-Square Statistic

We can also write the hypotheses in symbols as

H_0: $p_{blue} = 0.24$, $p_{orange} = 0.20$, $p_{green} = 0.16$, $p_{yellow} = 0.14$, $p_{red} = 0.13$, $p_{brown} = 0.13$,

H_a: At least one of the p_i s is incorrect

where p_{color} = the true population proportion of M&M S ® Milk Chocolate Candies of that color.

The idea of the chi-square goodness-of-fit test is this: we compare the **observed counts** from our sample with the counts that would be expected if H_0 is true.

The more the observed counts differ from the **expected counts**, the more evidence we have against the null hypothesis.

The Chi-Square Statistic

To see if the data give convincing evidence against the null hypothesis, we compare the observed counts to the expected counts assuming H_0 is true.

The statistic that we use to compare is called the chi-square statistic (or χ^2)

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

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Calculate the χ^2 value for your sample

Think of χ^2 as a measure of the distance of each observed count from the expected counts. Larger values give stronger evidence against H_0.

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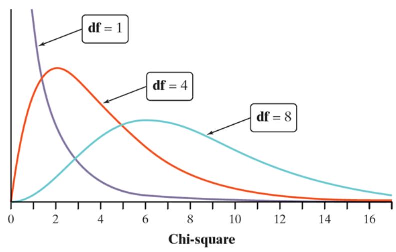
The Chi-Square Distributions and P-Values

The sampling distribution of the chi-square statistic is NOT Normal. It is right-skewed and is always a positive number.



Degrees of freedom (df) equal to the number of categories minus 1.

Expected counts must all be ≥ 5.

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Carrying Out a Test

Use the 4-step process:

STATE: state hypotheses and name the test

PLAN: Check conditions

Random: the data must come from a random sample or randomized experiment

10%: n ≤ (1/10) N

Large Counts: All EXPECTED counts are at least 5

(Note: Use COUNTS, not proportions!!)

DO: do the test

CONCLUDE: state your conclusion, include context, same as before: use α

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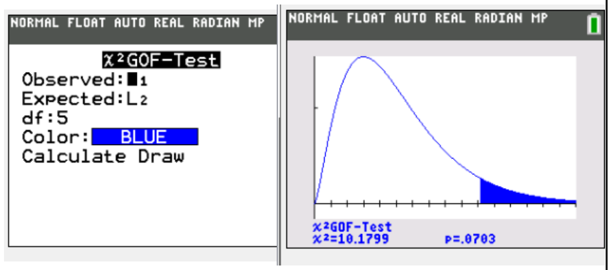
On the calculator:

Enter the observed counts in L1 and the expected counts in L2. Do not round the expected counts.

Perform the χ2 GOF test. df is the number of categories - 1.



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It is always a good idea to write the first few terms in the formula to show you know what you are doing:

$$\frac{(O_1 - E_1)^2}{E_1} +$$

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Follow up analysis:

When you run your test, the calculator creates a new list called CNTRB that shows how each observed value contributes to the chi-square statistic.

You can add this list to your lists, by selecting this list from the list of lists to add to the next available list.

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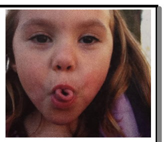
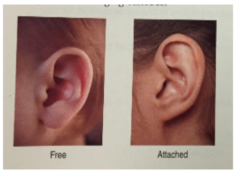
Some people can roll their tongues.

Some people have earlobes that are attached to their necks.





Tongue	Earlobes	Predicted Fraction
Non-curling	Attached	1/16
Non-curling	Free	3/16
Curling	Attached	3/16
Curling	Free	9/16

Genetic theory predicts that people will have neither, one or both of these traits in the ratios 1:3:3:9 per the above table.

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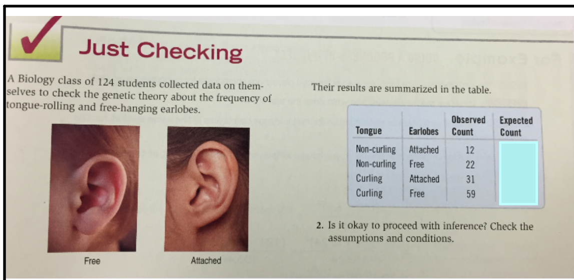

Just Checking

A Biology class of 124 students collected data on themselves to check the genetic theory about the frequency of tongue-rolling and free-hanging earlobes.

Free

Attached

Their results are summarized in the table.

Tongue	Earlobes	Observed Count	Expected Count
Non-curling	Attached	12	
Non-curling	Free	22	
Curling	Attached	31	
Curling	Free	59	

2. Is it okay to proceed with inference? Check the assumptions and conditions.

Do we have evidence to refute the claim about the ratios?

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